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Entwinement: a tool for bulk reconstruction in AdS_3/CFT_2

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[2211.17253 – Ben Craps, Marine De Clerck, AVL]

ULB / Solvay workshop
“Progress on gravitational physics: 45 years of Belgian-Chilean collaboration”

Intro: entanglement in holography

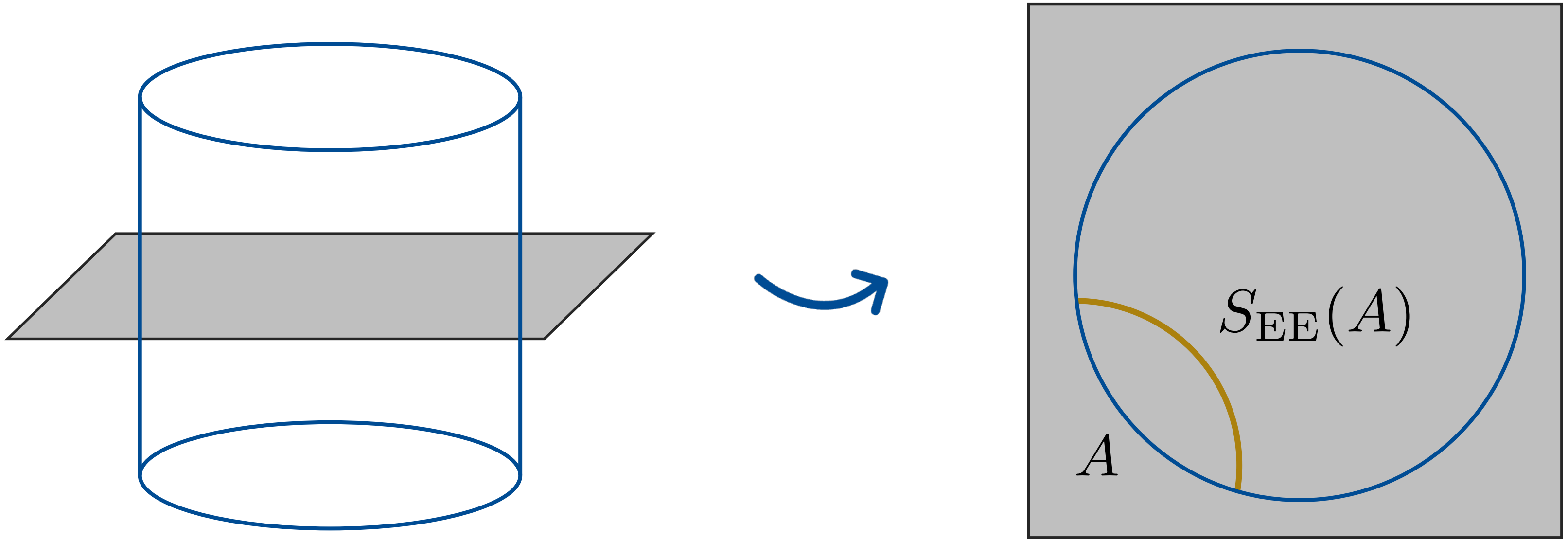
We have learned that gravity(1) should be understood as an **emergent** phenomenon from a more fundamental quantum-mechanical description.

But what is the mechanism?

(1) At least in AAdS

Intro: entanglement in holography

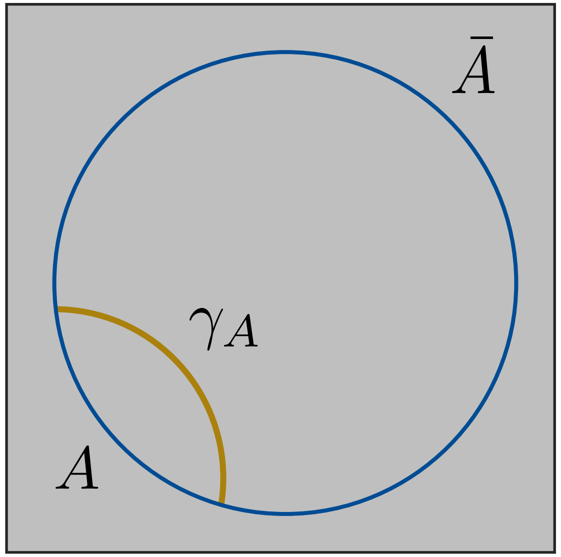
Entanglement and its dynamics in the fundamental quantum-mechanical description seem to be a key ingredient:



[0603001 – Ryu, Takayanagi]

Intro: entanglement in holography

Entanglement and its dynamics in the fundamental quantum-mechanical description seem to be a key ingredient:



- $\rho_A = \mathrm{Tr}_{\bar{A}} \left|\Psi\right\rangle \left\langle\Psi\right| = e^{-H_A}$
  $S_{\mathrm{EE}}(A) = -\mathrm{Tr}_A \rho_A \log \rho_A = \mathrm{length}(\gamma_A)/(4G_N)$
- 1st law of entanglement (perturb state) $\delta S_{\mathrm{EE}}(A) = \delta \left\langle H_A \right\rangle$
- In the bulk, for a ball-shaped boundary region:

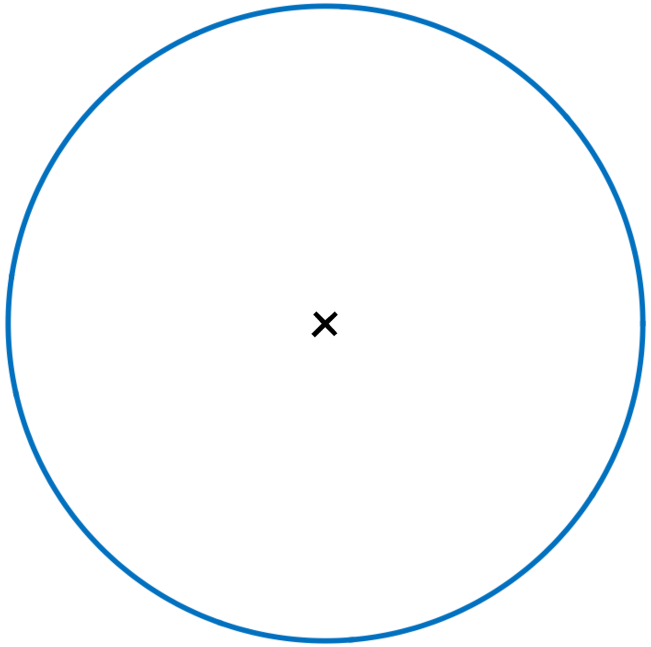
$$\delta S(A) \sim \int_{\gamma_A} \delta g_{ij} \leftrightarrows \delta \left\langle H_A \right\rangle \sim \int_A \delta g_{tt}$$

This implies bulk linearized Einstein equations!

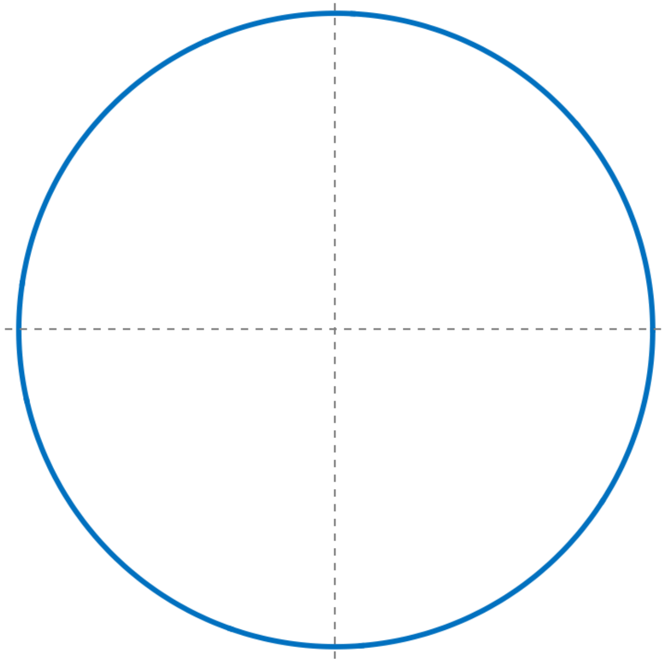
[1308.3716 – Lashkari, McDermott, Van Raamsdonk]
[1312.7856 – Faulkner, Guica, Hartman, Myers, Van Raamsdonk]

Introduction: conical defects

Conical defects arise from quotienting AdS$_3$



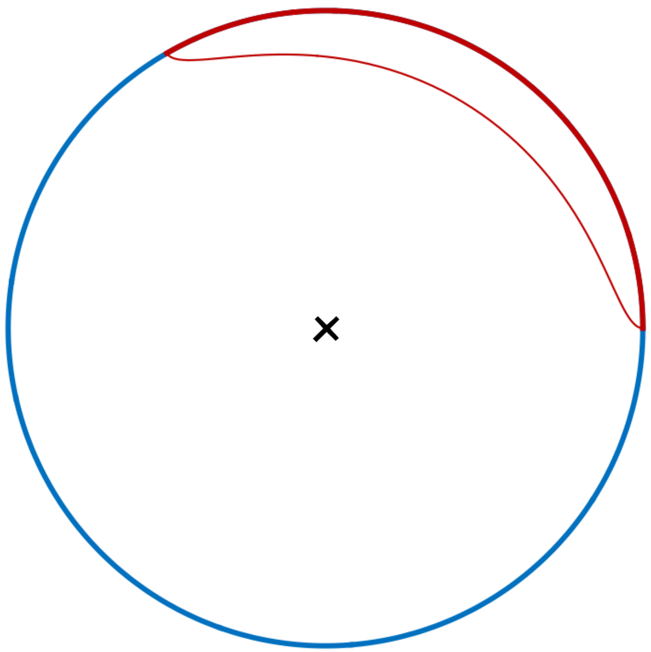
$\mathrm{AdS}_3/\mathbb{Z}_n \quad [n=4]$



AdS_3

Introduction: conical defects

RT surfaces (geodesics) can be computed from the covering



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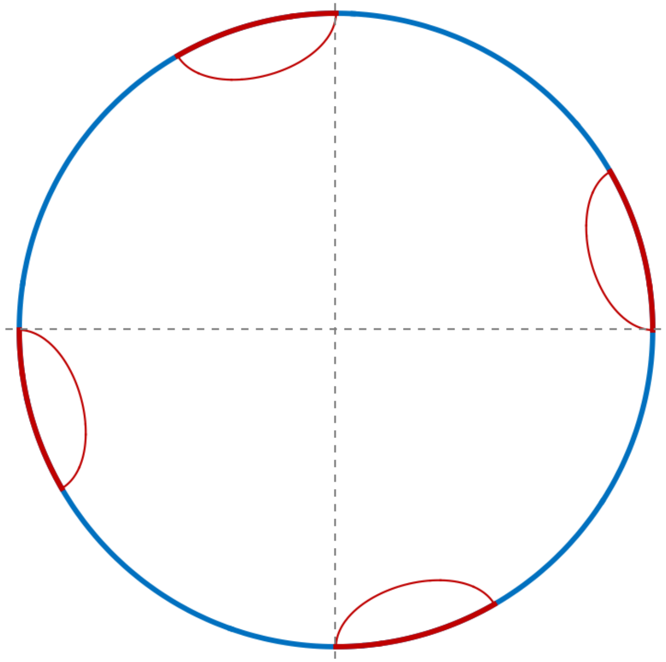

AdS_3

Introduction: conical defects

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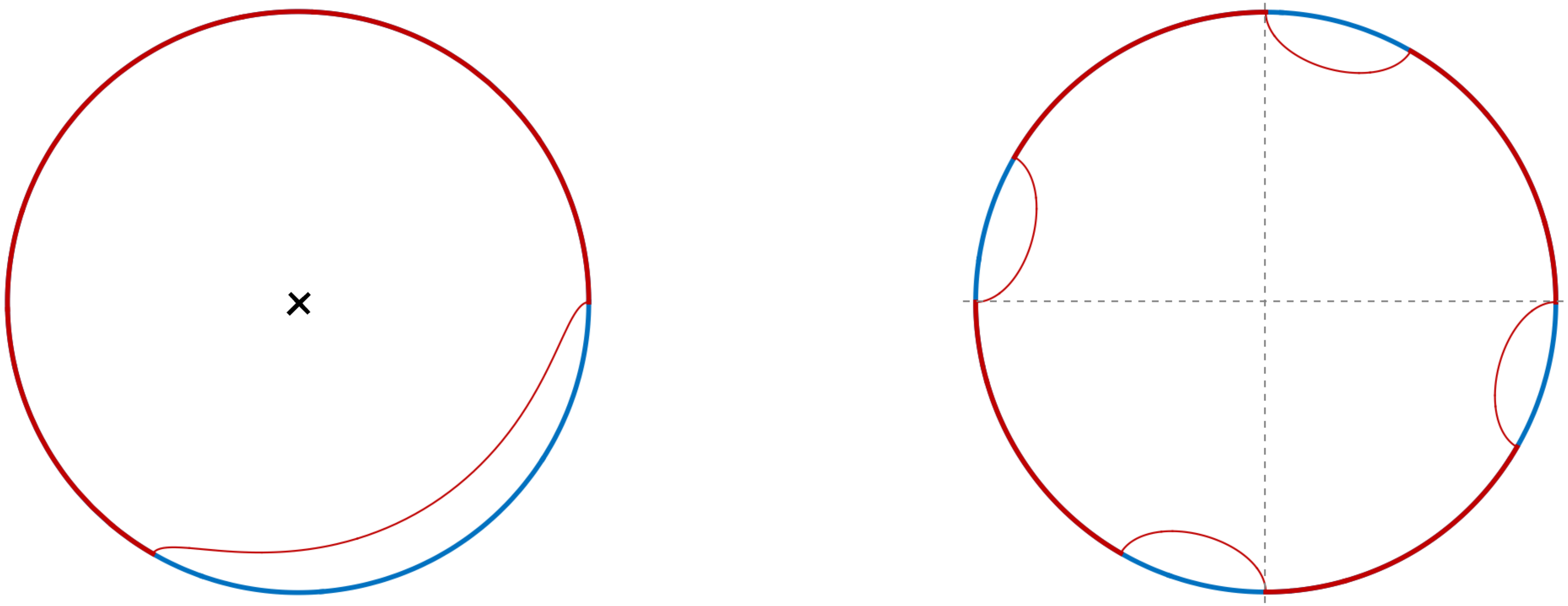
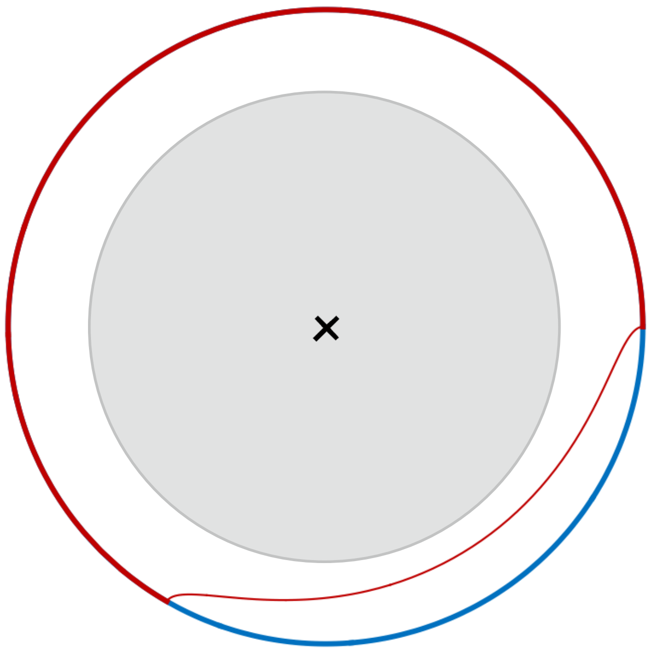


$\mathrm{AdS}_3/\mathbb{Z}_n \quad [n=4]$

AdS_3

Introduction: conical defects

There is a shadow for RT geodesics!



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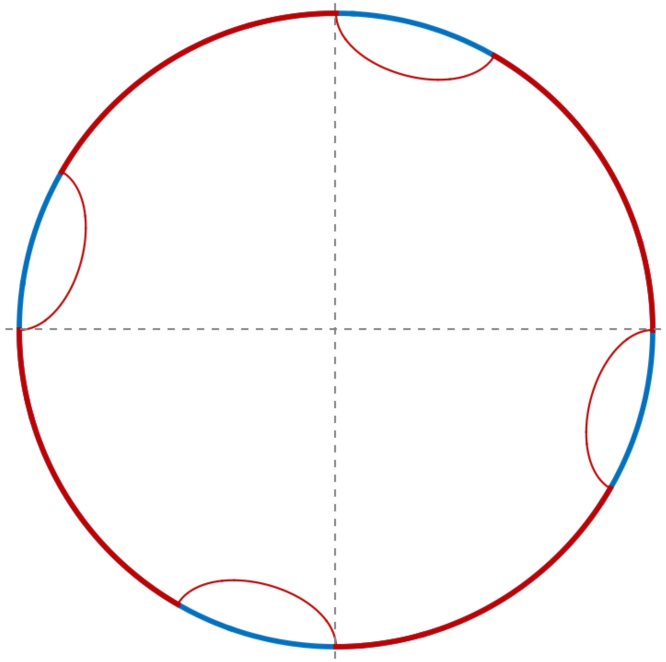

AdS_3

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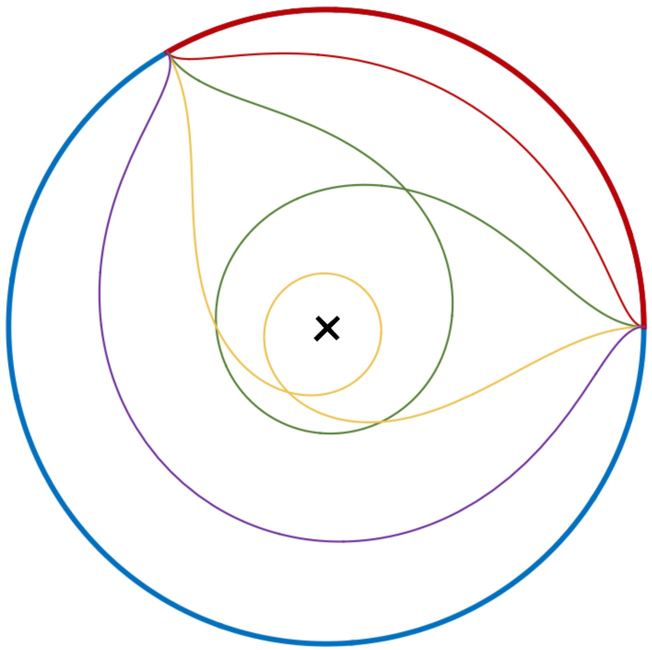
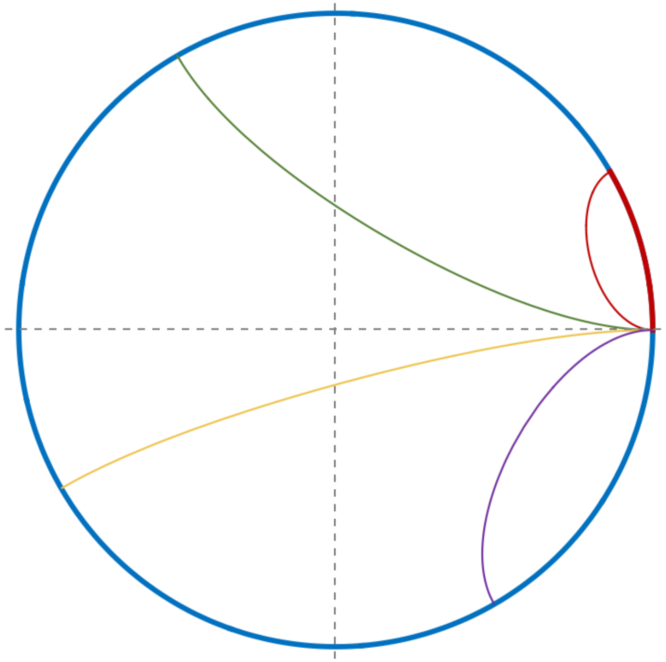
Introduction: conical defects

There are also **extremal** but **non-minimal** curves which can wind. What do they represent?



$\mathrm{AdS}_3/\mathbb{Z}_n \quad [n=4]$



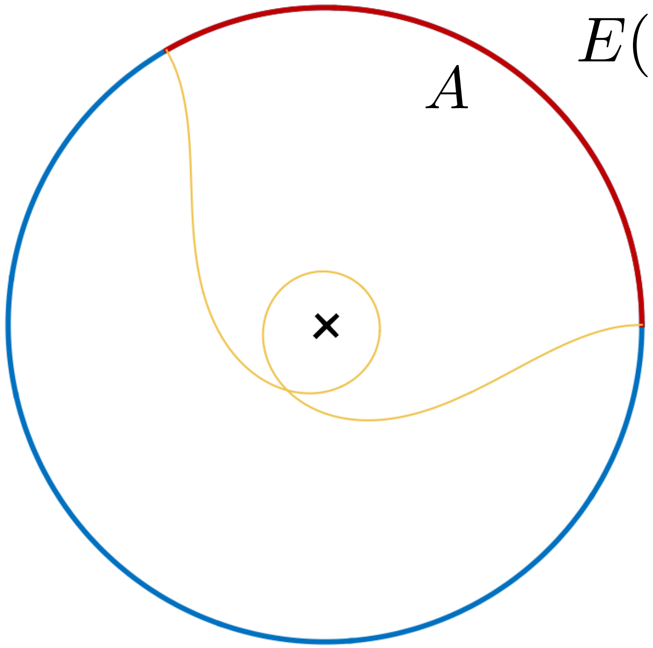
AdS_3

Introduction: conical defects

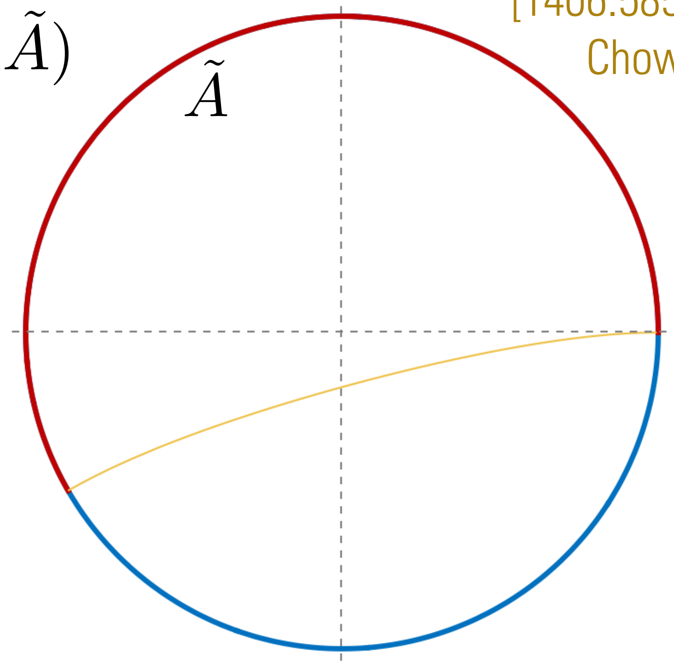
Entwinement was the name given to what the length of these geodesics measure

[1406.5859 – Balasubramanian, Chowdhury, Czech, de Boer]

$$E(A) = \sum_{g\in\mathbb{Z}_n} S(g\tilde{A})$$



$\mathrm{AdS}_3/\mathbb{Z}_n \quad [n=4]$



AdS_3

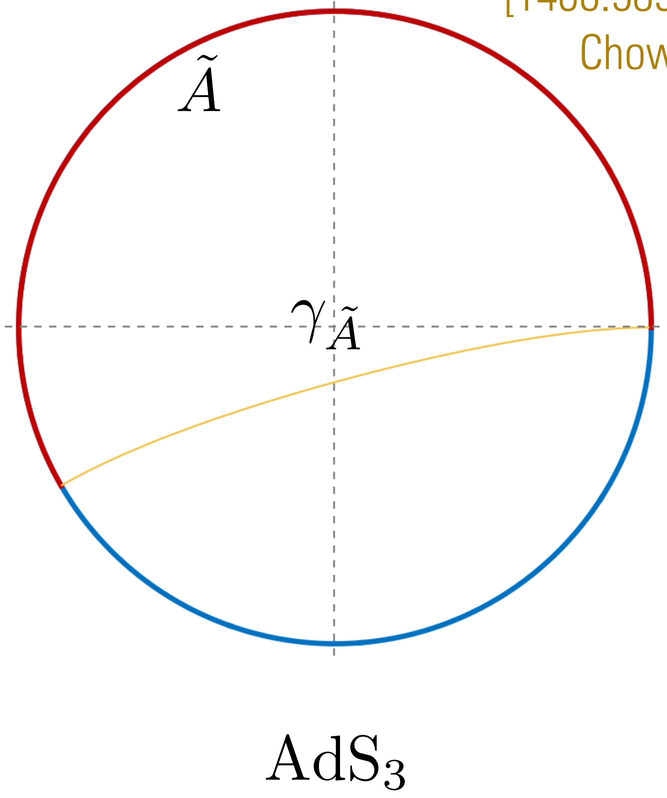
Introduction: conical defects

Entwinement was the name given to what the length of these geodesics measure

[1406.5859 – Balasubramanian, Chowdhury, Czech, de Boer]

$$E(A) = \sum_{g \in \mathbb{Z}_n} S(g\tilde{A}) = n\frac{\text{length}(\gamma_{\tilde{A}})}{4\tilde{G}_N}$$

This quantity allows to reconstruct the geometry deeper in the bulk than spatial entanglement entropy



Introduction: entwinement

There has been some confusion trying to define entwinement **directly from the boundary**:

- Originally, defined in terms of a covering theory (analogous to the bulk construction)

  [1406.5859 – Balasubramanian, Chowdhury, Czech, de Boer]

- Defined as algebraic EE in a toy model

  [1608.02040 – Jennifer Lin] ✗

- Defined from a replica trick construction, and as the vN-entropy of a reduced density matrix

  [1609.03991 – Balasubramanian, Bernamonti, Craps, De Jonckheere, Galli]

  [1806.02871 – Balasubramanian, Craps, De Jonckheere, Sarosi] ?

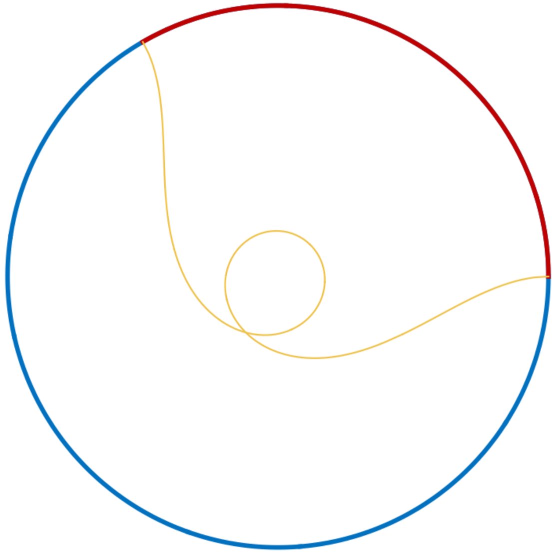
- Defined formally as a minimum of a set of vN-entropies after projecting the state

  [1910.05352 – Erdmenger, Gerbershagen] ✓

Introduction: entwinement

Why do we care?

- Evidently, **entwinement is not conventional spatial EE of a subregion.**
- It will turn out to measure some notion of entanglement between an **internal set of degrees of freedom.**
- Is this interesting more generally in holography?



Outline

1. An $\mathrm{AdS}_3/\mathrm{CFT}_2$ primer
2. Symmetric product orbifolds on the lattice
3. A detour: entanglement between identical particles
4. Entanglement and entwinement in symmetric product orbifolds
5. Comments, thoughts and conclusions

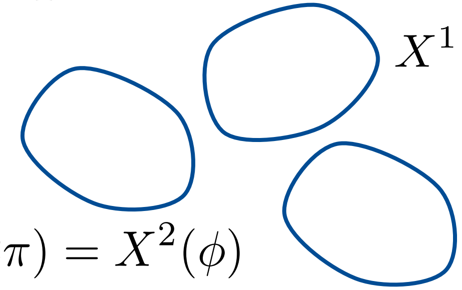
An AdS_3/CFT_2 primer

Weakly coupled strings in AdS_3 x S^3 x T^4 ⇄ 2-dimensional (S)CFT

Deformation @ strong coupling of a
free sigma model with target $(T_4)^N/S_N$

$$S = \sum_{a=1}^{N} \sum_{i=1}^{D} \int \mathrm{d}t \, \mathrm{d}\phi \, \partial_\mu X^{i,a} \partial^\mu X^{i,a} + \ldots$$

before S_N quotient



$$X^1(\phi + 2\pi) = X^1(\phi)$$

$$X^2(\phi + 2\pi) = X^2(\phi)$$

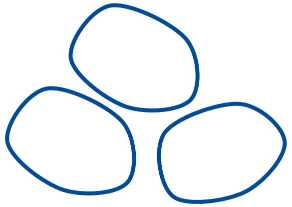
$$X^3(\phi + 2\pi) = X^3(\phi)$$

An AdS$_3$/CFT$_2$ primer

$$S = \sum_{a=1}^{N} \int \mathrm{d}t\, \mathrm{d}\phi\, \partial_\mu X^a \partial^\mu X^a + \ldots$$

S$_N$ identification allows non-trivial boundary conditions (twisted sectors)

- Empty AdS$_3$ described by the trivial sector $X^a(\phi + 2\pi) = X^a(\phi)$



- For the Z$_n$ conical defect, partition the N **strands** into N/n **long strings** of length n

[0011217 – Balasubramanian, de Boer, Keski-Vakkuri, Ross]

[0106171 / 0206175 – Martinec, McElgin]

[0508110 – Balasubramanian, Kraus, Shigemori]

$$X^1 \xrightarrow{2\pi} X^2 \xrightarrow{2\pi} \ldots \xrightarrow{2\pi} X^n \xrightarrow{2\pi} X^1$$

$$X^{n+1} \xrightarrow{2\pi} X^{n+2} \xrightarrow{2\pi} \ldots \xrightarrow{2\pi} X^{2n} \xrightarrow{2\pi} X^{n+1}$$

$$\vdots$$

An AdS$_3$/CFT$_2$ primer

$$S = \sum_{a=1}^{N} \int \mathrm{d}t\, \mathrm{d}\phi\, \partial_\mu X^a \partial^\mu X^a + \dots$$

S$_N$ identification allows non-trivial boundary conditions (twisted sectors)

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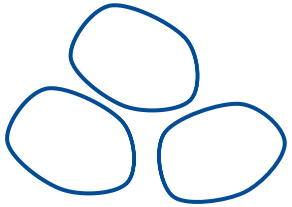


- For the Z$_n$ conical defect, partition the N **strands** into N/n **long strings** of length n

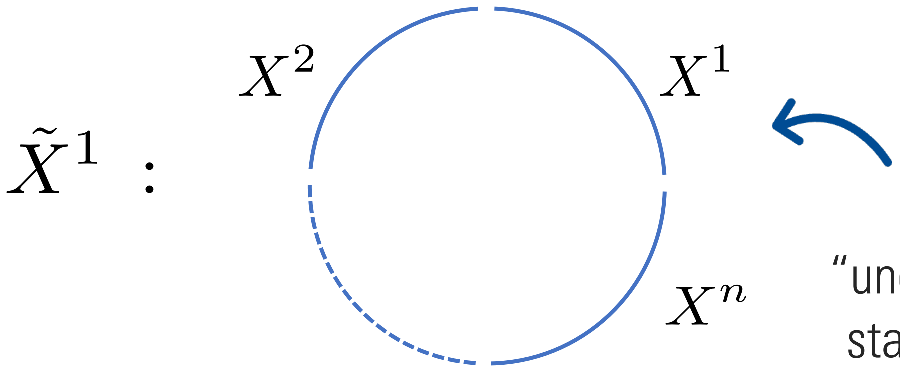
Keep in mind:

- S$_N$ quotient is part of the definition of the target space of the dual CFT.
- The Z$_n$ conical defect geometry corresponds to a certain state(s) in that S$_N$ quotiented CFT.

An AdS$_3$/CFT$_2$ primer

$$S = \sum_{a=1}^{N} \int \mathrm{d}t\, \mathrm{d}\phi\, \partial_\mu X^a \partial^\mu X^a + \ldots \longrightarrow S = \sum_{\tilde{a}=1}^{N/n} \int \mathrm{d}t\, \mathrm{d}\phi\, \partial_\mu \tilde{X}^{\tilde{a}} \partial^\mu \tilde{X}^{\tilde{a}} + \ldots$$

It is possible to work in an equivalent theory of N/n long strings on an n-times longer circle



S$_{N/n}$ orbifold CFT, we have "ungauged" a Z$_n$ for the particular states dual to the conical defect

Certain quantities (SUSY-protected or computed in the theory of long strings) are expected to be reliably computed at the free orbifold point.

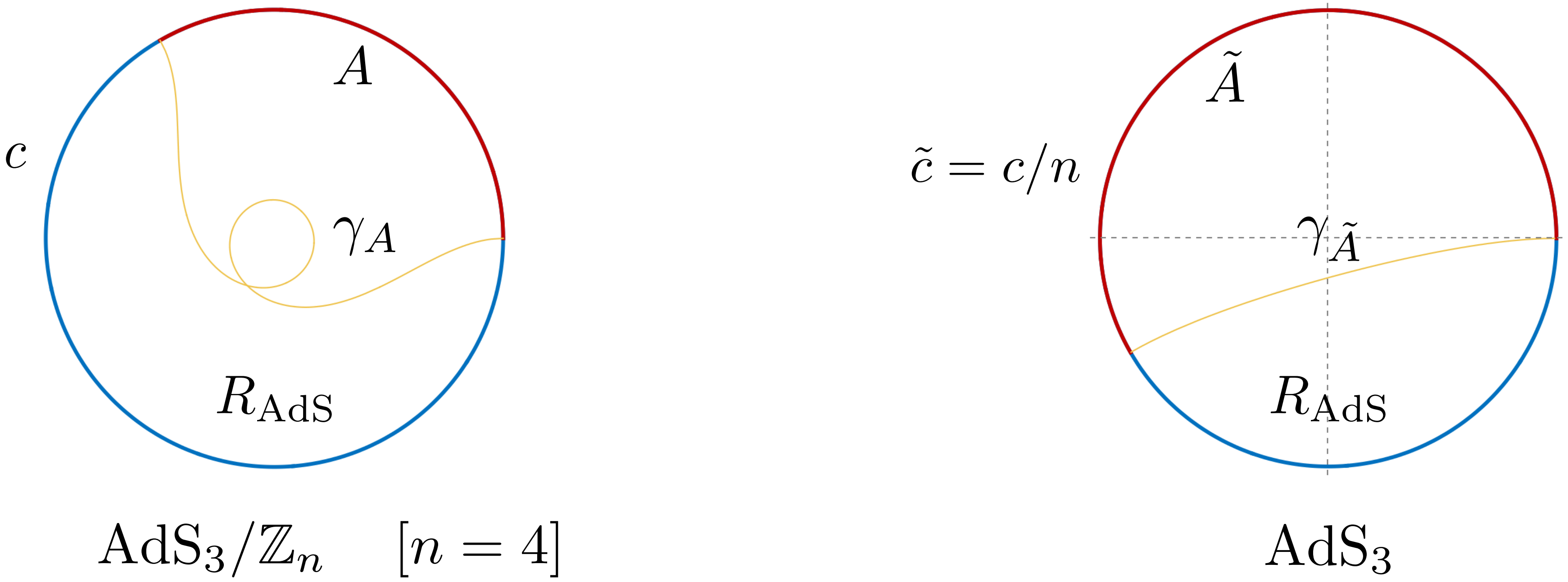
[0106171 / 0206175 – Martinec, McElgin]

[1609.03991 – Balasubramanian, Bernamonti, Craps, De Jonckheere, Galli]

An AdS$_3$/CFT$_2$ primer

In the original work, **entwinement** was defined as spatial EE in the theory of long strings

$$E(A) = \sum_{g\in\mathbb{Z}_n} S(g\tilde{A}) = n\frac{\text{length}(\gamma_{\tilde{A}})}{4\tilde{G}_N} = \frac{\text{length}(\gamma_A)}{4G_N}$$



An AdS$_3$/CFT$_2$ primer

$$E(A) = \sum_{g\in\mathbb{Z}_n} S(g\tilde{A}) = n\frac{\text{length}(\gamma_{\tilde{A}})}{4\tilde{G}_N} = \frac{\text{length}(\gamma_A)}{4G_N}$$

- Defined in this way, entwinement respects the original Z$_n$ (gauge) symmetry.
- It is not completely clear what it is computing from the viewpoint of the original theory.

Goal: define measures of entanglement between internal degrees of freedom in symmetric product orbifold theories.

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SPOs on the lattice

Basic ingredients of symmetric product orbifolds

- N copies of the field $\hat{g}_i X_i^a \hat{g}_i^{-1} = X_i^{g_i(a)} \longrightarrow \hat{g}_i \mathbf{X}_i \hat{g}_i^{-1} = g_i^{-1}\mathbf{X}_i$ $\quad [i = 1, \dots, L]$

  Non-derivative terms are S_N-invariant $\quad \hat{g}_i \mathbf{X}_i \cdot \mathbf{X}_i \hat{g}_i^{-1} = \mathbf{X}_i \cdot \mathbf{X}_i$

- Background S_N gauge field $\quad \hat{g}_i U_{i+1,i} \hat{g}_i^{-1} = U_{i+1,i} g_i \qquad \hat{g}_{i+1} U_{i+1,i} \hat{g}_{i+1}^{-1} = g_{i+1}^{-1} U_{i+1,i}$

  Nearest-neighbor couplings S_N-invariant $\quad (\mathbf{X}_{i+1} - U_{i+1,i}\mathbf{X}_i)^T (\mathbf{X}_{i+1} - U_{i+1,i}\mathbf{X}_i)$

- Hilbert space $\quad |\{\mathbf{x}_j\}, \{u_{j+1,j}\}\rangle$

$$\hat{g}_i \, |\{\mathbf{x}_j\}, \{u_{j+1,j}\}\rangle = |\{g_i \mathbf{x}_i, \mathbf{x}_j\}_{j \neq i}, \{u_{i+1,i} g_i^{-1}, g_i u_{i,i-1}, u_{j+1,j}\}_{j \neq i, i-1}\rangle$$

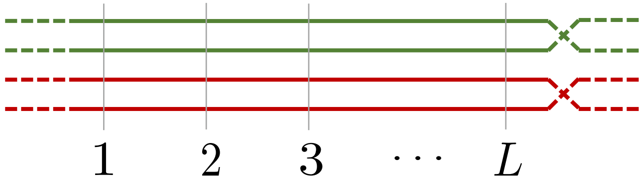
SPOs on the lattice

Now project into the subspace carrying the trivial representation of S$_N$ ("gauge S$_N$")

$$\hat{P} = \bigotimes_{i=1}^{L} \left(\frac{1}{N!} \sum_{g_i \in S_N} \hat{g}_i \right) = \left[\frac{1}{N!} \sum_{g \in S_N} \hat{g} \otimes \ldots \otimes \hat{g} \right] \left[\bigotimes_{i=1}^{L-1} \left(\frac{1}{N!} \sum_{g_i \in S_N} \hat{g}_i \right) \right] \equiv \hat{P}_{gl} \hat{P}_{L-1}$$

Partially gauged theory ("global states") $\mathcal{H}_{gl} = \hat{P}_{L-1} \mathcal{H}$

$$|\{\mathbf{x}_i\}, u\rangle_{gl} \equiv \sqrt{(N!)^{L-1}} \hat{P}_{L-1} \, |\{\mathbf{x}_i\}, \{1, \ldots, 1, u\}\rangle$$



$u = (1\,2)(3\,4)$

Residual symmetry: $(\hat{g} \otimes \cdots \otimes \hat{g}) \, |\{\mathbf{x}_i\}, u\rangle_{gl} = |\{g\mathbf{x}_i\}, gug^{-1}\rangle_{gl}$

SPOs on the lattice

Now project into the subspace carrying the trivial representation of S$_N$ (“gauge S$_N$”)

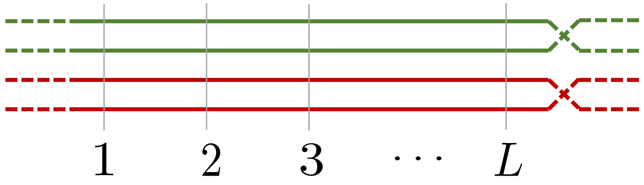
SPO theory (“symmetric states”) $\mathcal{H}_S = \hat{P}_{gl}\;\mathcal{H}_{gl} = \text{span}\left\{|\{\mathbf{x}_i\},u\rangle_S \;|\; \mathbf{x}_i \in \mathcal{M}^N, u \in \mathcal{C}(S_N)\right\}$

$$|\{\mathbf{x}_i\},u\rangle_S \equiv \sqrt{N!}\hat{P}_{gl}\,|\{\mathbf{x}_i\},u\rangle_{gl} = \frac{1}{\sqrt{N!}}\sum_{g\in S_N}|\{g\mathbf{x}_i\},gug^{-1}\rangle_{gl}$$

Only conjugacy class matters
(twisted sectors)

Redundancy: $|\{\mathbf{x}_i\},u\rangle_S = |\{h\mathbf{x}_i\},u\rangle_S \quad \forall h \in C_u$

$$|\Psi_u\rangle = \frac{1}{\sqrt{|C_u|}}\int_{\{\mathbf{x}_i\}}\Psi_u(\{\mathbf{x}_i\})\,|\{\mathbf{x}_i\},u\rangle_S \longrightarrow \Psi_u(\{h\mathbf{x}_i\}) = \Psi_u(\{\mathbf{x}_i\})$$



$u = (1\,2)(3\,4)$

$$[u] = \{(1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$$

$$C_u = \{1, (1\,2), (3\,4), (1\,3)(2\,4), (1\,2)(3\,4), (1\,3\,2\,4), (1\,4\,2\,3), (1\,4)(2\,3)\}$$

Entanglement & identical particles

The previous story is very similar to what happens with identical particles in QM

- Symmetric states (bosonic) $\mathcal{H}_S = \text{span}\,\{|x_1,\dots,x_N\rangle_S\,\}$

$$|x_1,x_2,\dots,x_N\rangle_S = \frac{1}{\sqrt{N!}}\sum_{g\in S_N}|x_{g(1)},x_{g(2)},\dots,x_{g(N)}\rangle$$

There is no notion of connectedness now (identity twisted sector)

Redundancy: $|x_1,x_2,\dots,x_N\rangle_S = |x_{g(1)},x_{g(2)},\dots,x_{g(N)}\rangle_S \quad \forall g\in S_N$

$$|\psi_S\rangle = \frac{1}{\sqrt{N!}}\int_{x_a}\psi_S(x_1,\dots,x_N)\,|x_1,\dots,x_N\rangle_S \longrightarrow \psi_S(x_1,\dots,x_N)=\psi_S(x_{g(1)},\dots,x_{g(N)})$$

Entanglement & identical particles

We want to quantify entanglement between k particles and the remaining N-k

- If the k-particle operators formed an algebra:

$$\mathrm{Tr}_{\mathcal{H}_S}\left[\rho_{(k)}\mathcal{O}^{(k)}\right] \equiv \langle\psi_S|\mathcal{O}^{(k)}|\;\psi_S\rangle \quad\longrightarrow\quad S_{\mathrm{vN}}(\rho_{(k)}) = -\mathrm{Tr}_{\mathcal{H}_S}\left[\rho_{(k)}\;\log\rho_{(k)}\right]$$

- But they do not form an algebra:

$$\mathcal{O}^{(k)} = \frac{1}{N!}\int \mathcal{O}_k(x_1,\ldots,x_k;y_1,\ldots,y_k)\,|x_1,\ldots,x_k,z_{k+1},\ldots,z_N\rangle_S\ {}_S\langle y_1,\ldots,y_k,z_{k+1},\ldots,z_N|$$

It is only a linear space closed under conjugation.

In each term within the state
the operator acts on a different
set of k particles

Entanglement & identical particles

We can still compute expectation values from something like a reduced density matrix:

$$\langle\psi_S|\mathcal{O}^{(k)}|\psi_S\rangle = \int \mathcal{O}_k(x_1,\ldots,x_k;y_1,\ldots,y_k)\rho_{(k)}(y_1,\ldots,y_k;x_1,\ldots,x_k)$$

$$\rho_{(k)}(x_1,\ldots,x_k;x'_1,\ldots,x'_k) = \int_{y_a} \psi_S(x_1,\ldots,x_k,y_{k+1},\ldots,y_N)\psi^*_S(x'_1,\ldots,x'_k,y_{k+1},\ldots,y_N)$$

We think of this as an operator on an auxiliary k-particle space and compute:

$$S_k \equiv S_{\rm vN}(\rho_{(k)}) = -\mathrm{Tr}_{\mathcal{H}_k}\left[\rho_{(k)}\ \log\rho_{(k)}\right]$$

Entanglement & entwinement in SPOs

Pick a subset A of vertex degrees of freedom and consider the operators:

$$\begin{aligned}\mathcal{O}^{(A)} &= \frac{1}{|C_u|}\int_{\mathbf{x},\mathbf{y}} \mathcal{O}_A(\mathbf{y}_A;\mathbf{x}_A)\delta(\mathbf{y}_{\bar{A}}-\mathbf{x}_{\bar{A}})\,|\mathbf{y},u\rangle_S\;{}_S\langle\mathbf{x},u| \\ &= \frac{1}{|C_u|^2}\int_{\mathbf{x},\mathbf{y}} \left(\sum_{h\in C_u} \mathcal{O}_A((h\mathbf{y})_A;(h\mathbf{x})_A)\delta((h\mathbf{y})_{\bar{A}}-(h\mathbf{x})_{\bar{A}})\right)|\mathbf{y},u\rangle_S\;{}_S\langle\mathbf{x},u|\end{aligned}$$

Connectedness of A
defined relative to u

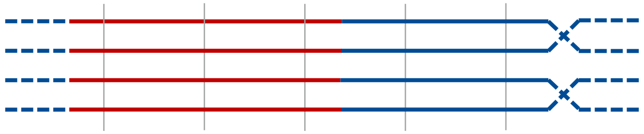
Whether or not this is an algebra depends on A.

Entanglement & entwinement in SPOs

Whether or not this is an algebra depends on A:

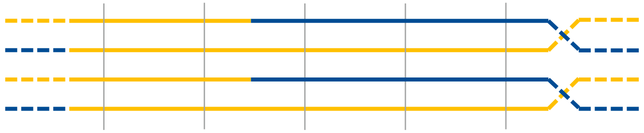
$$\mathcal{O}^{(A)}\mathcal{Q}^{(A)} = \frac{1}{|C_u|^2}\int_{\mathbf{x},\mathbf{y},\mathbf{z}}\sum_{h\in C_u}\mathcal{O}_A(\mathbf{y}_A;(h\mathbf{z})_A)\mathcal{Q}_A(\mathbf{z}_A;\mathbf{x}_A)\delta(\mathbf{y}_{\bar{A}}-(h\mathbf{z})_{\bar{A}})\delta(\mathbf{z}_{\bar{A}}-\mathbf{x}_{\bar{A}})\,|\mathbf{y},u\rangle_S\ {}_S\langle\mathbf{x},u|$$

- For a spatial partition, they form an algebra (as expected)



$$(h\mathbf{x})_A\cap(\mathbf{x})_{\bar{A}}=\emptyset \longrightarrow \mathcal{O}_A\left((h\mathbf{x})_A;(h\mathbf{y})_A\right)=\mathcal{O}_A\left(\mathbf{x}_A;\mathbf{y}_A\right)$$

- For partitions which are not invariant under the centralizer C_u, it is not an algebra



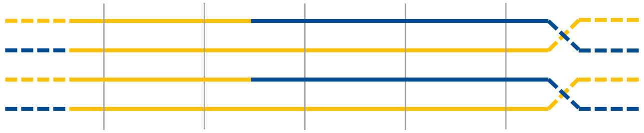
Entanglement & entwinement in SPOs

We can also compute expectation values of operators supported on A by means of a reduced density matrix:

$$\langle\Psi_u|\mathcal{O}^{(A)}|\Psi_u\rangle = \int_{\mathbf{x}_A,\mathbf{y}_A} \mathcal{O}_A(\mathbf{y}_A;\mathbf{x}_A)\rho_A(\mathbf{x}_A;\mathbf{y}_A) \qquad \rho_A(\mathbf{x}_A,\mathbf{x}'_A) = \int_{\mathbf{y}_{\bar{A}}} \Psi_u(\mathbf{x}_A,\mathbf{y}_{\bar{A}})\Psi^\star_u(\mathbf{x}'_A,\mathbf{y}_{\bar{A}})$$

Entwinement is defined then as the von Neumann entropy of this density matrix, for A the union of N/n identical connected pieces, one in each long string:

$$E(A) = S_{\mathrm{vN}}(\rho_A) = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$



$$u = (1\,2\ldots n)(n+1\ldots 2n)\ldots(N-n+1\ldots N)$$

Entanglement & entwinement in SPOs

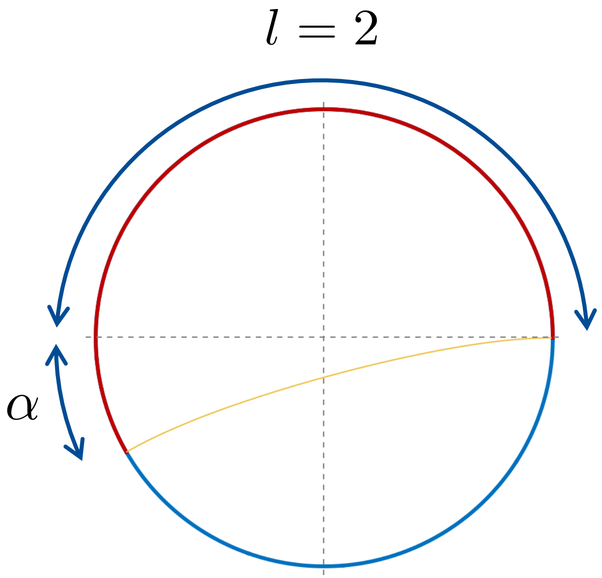
We can check that this definition matches the one obtained from the lengths of non-minimal geodesics:

$$u = (1\,2\ldots n)(n+1\ldots 2n)\ldots(N-n+1\ldots N)$$

$$\Psi_u(\mathbf{x}) = \psi_0(\mathbf{x}_{LS_1})\ldots\psi_0(\mathbf{x}_{LS_{N/n}})$$

$$S_{\mathrm{vN}}\left(\rho_{l,\alpha}\right) = \frac{N}{n}\frac{c}{3(N/n)}\log\left[\frac{2nR_{\mathrm{AdS}}}{\epsilon_{\mathrm{UV}}}\sin\left(\frac{\alpha+2\pi l}{2n}\right)\right]$$

$$E_{l,\alpha} \equiv S_{\mathrm{vN}}\left(\rho_{l,\alpha}\right) = \mathcal{L}_\ell(\alpha)/(4G_N)$$



Entanglement & entwinement in SPOs

We can now revisit the previous definitions of entwinement:

- Originally, defined in terms of a covering theory (analogous to the bulk construction)

  [1406.5859 – Balasubramanian, Chowdhury, Czech, de Boer]

- Defined as algebraic EE in a toy model ✗

  [1608.02040 – Jennifer Lin]

- Defined from a replica trick construction, and as the vN-entropy of a reduced density matrix ?

  [1609.03991 – Balasubramanian, Bernamonti, Craps, De Jonckheere, Galli]

  [1806.02871 – Balasubramanian, Craps, De Jonckheere, Sarosi]

- Defined formally as a minimum of a set of vN-entropies after projecting the state ✓

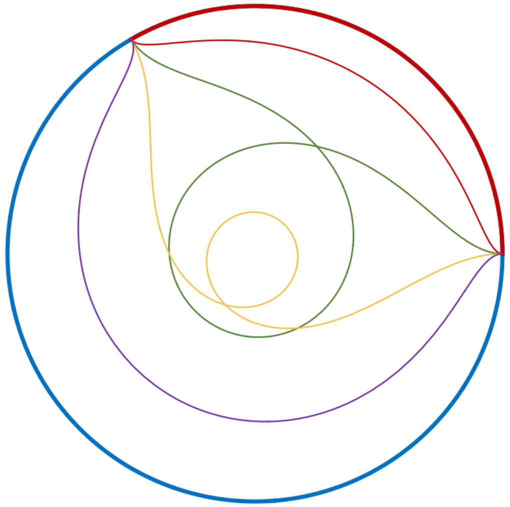
  [1910.05352 – Erdmenger, Gerbershagen]

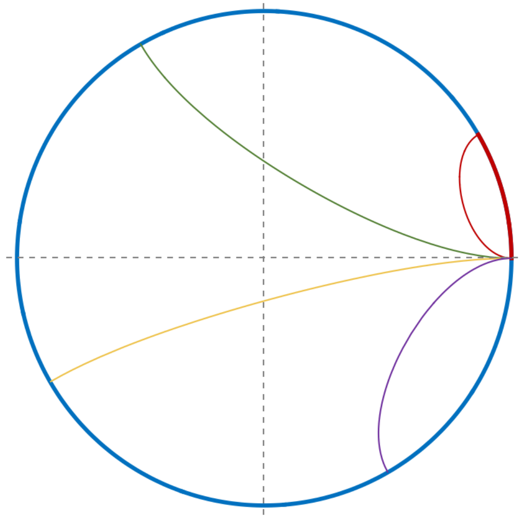
Comments, thoughts and conclusions

The conical defects offer a nice setup for discussions close in spirit to other works:

- Given that we have several extremal curves anchored to A… Do we have a "**python**"? Is this telling us something about **how hard it is to reconstruct** the geometry between the outermost extremal curve and the true RT geodesic?

[1912.00228 – Brown, Gharibyan, Penington, Susskind]





In our dual definition of entwinement, we need operators not supported on a subregion

Comments, thoughts and conclusions

- We have proposed a way to quantify entanglement between internal degrees of freedom in symmetric product orbifold CFTs.
- For a particular partition of the degrees of freedom, one recovers the notion of **entwinement**, relevant in holographic setups with conical defects.
- This was in a sense a very simple model of a more broad (and potentially interesting) question. Is **entanglement between internal** (often gauged) **degrees of freedom** important to understand the **emergence of a geometric, gravitating picture**?

¡Muchas gracias! / Merci beaucoup!

Backup: AdS_3/CFT_2

[1012.0072– Avery]

SUGRA picture (IIB): $1 \ll 1/g_s \ll N_1, N_5$

$$\left(\frac{H_1}{H_5}\right)^{1/4} \mathrm{d}s_E^2 = \frac{1}{\sqrt{H_1 H_5}}\left(-\mathrm{d}t^2 + \mathrm{d}y^2\right) + \sqrt{H_1 H_5}\,\underbrace{\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_3^2\right)}_{\text{4D transverse space}} + \sqrt{V_{T^4}\frac{H_1}{H_5}}\,\mathrm{d}s_{T^4}^2$$

Large S_1

Small T^4

$C^{(2)}$ turned on

$$H_{1/5} = 1 + \frac{Q_{1/5}}{r^2}$$

$$Q_1 = \frac{g_s^\infty \ell_s^2}{v_\infty} N_1$$

$$Q_5 = g_s^\infty \ell_s^2 N_5$$

- N_1 D1-branes around y, N_5 D5-branes around y and T^4.
- It is ¼-BPS (8 supercharges).
- The near-horizon limit (small r) is AdS_3 x S^3 x T^4, Q_1Q_5 sets the size.
- 20-dim moduli space in the near-horizon limit.

$$\frac{SO(5,4)}{SO(5) \times SO(4)}$$

Backup: AdS$_3$/CFT$_2$

[1012.0072– Avery]

Brane picture: $1 \ll N_1, N_5 \ll 1/g_s$

1. Standard worldvolume gauge theory of the D1/D5, reduced to 2d:

   - Open strings D1/D1, D1/D5, D5/D5 give a SUSY U(N$_1$) x U(N$_5$) gauge theory with a certain potential.
   - At low energies, the moduli space V=0 becomes the target space of the IR CFT.

     [Technically, we look at the Higgs branch where the branes are not separated.]
   - The resulting theory has 8 supercharges, c = 6(N$_1$N$_5$ + 1), and gauge group S$_{N5}$ x S$_{N1}$. Its moduli space (marginal operators) coincides with the SUGRA description (20-dim).

This is not believed to be the best picture of the dual, although it has many right ingredients.

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Backup: AdS$_3$/CFT$_2$

[1012.0072– Avery]

Brane picture: $1 \ll N_1, N_5 \ll 1/g_s$

② Instanton description: [9512077– Douglas // 9512078 – Vafa // 9707093 – Witten]

- In the D5 U(N$_5$) worldvolume theory, one can define instantons with self-dual field strength with respect to T^4. They source C$^{(2)}$. They are interpreted as D1s wrapping S^1!
- The IR CFT captures the zero modes of these instantons: it is a 2d N=(4,4) sigma model with target space the space of zero modes. It has c = 6 N$_1$N$_5$, and a 20-dim space of marginal deformations (like SUGRA moduli).
- This target space is a (smooth deformation of) $\frac{(T^4)^{N_1 N_5}}{S_{N_1 N_5}}$

For N$_5$=1, N$_1$ equivalent instantons for which we specify the position on T^4

- There is a point in moduli space (orbifold) where the target space is exactly $\frac{(T^4)^{N_1 N_5}}{S_{N_1 N_5}}$